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LETTER TO THE EDITOR

The magnetization process in type-II superconducting film

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Abstract. We numerically solve the time-dependent Ginzburg–Landau equation for a homogeneous, isotropic type-II superconducting film in the presence of an external magnetic field. We study the dynamical processes of the magnetization by stepwise change of the external field. It is demonstrated that the magnetic vortices penetrate and are expelled from the system boundary in accord with the change of the field, and the dynamical instability of magnetic flux wall near the boundary leads to the formation of a vortex at low fields. The hysteresis loop of magnetization versus the external field is also obtained.

The time-dependent Ginzburg–Landau (TDGL) equation is a useful starting equation in studying the dynamics of superconductors [1]. There appear to have been several numerical attempts to study stationary equilibrium states (not dynamical processes) for superconductors as functions of external field and/or temperature by using various non-linear optimization methods for the Ginzburg–Landau (GL) equation [2, 3] and London equation [4], as well as some analytic study [1]. However, despite its importance, little study of the TDGL equation has been done either analytically or numerically, due to its high degree of non-linearity and/or the coupling between the superconducting order parameter and vector potential. Recently, several authors have independently proposed numerical methods to solve the TDGL equation for various physical situations [5–8]. In this letter, using our previously described simulation method [5, 8], we briefly report the dynamical behaviour of the magnetization process on changing the external magnetic field. We particularly focus on the visualization of such dynamical processes, since little study from this point of view has been done.

In the standard way we write the homogeneous, isotropic GL free energy functional $F[\Phi, \boldsymbol{A}]$ with order parameter $\Phi(\boldsymbol{r}, t)$ and vector potential $\boldsymbol{A}(\boldsymbol{r}, t)$ at time t in position $\boldsymbol{r}$ as [1]

$$F = \int \mathrm{d}\boldsymbol{r}\ \left[\frac{1}{2m}|D\Phi|^2 + \alpha|\Phi|^2 + \frac{\beta}{2}|\Phi|^4 + \frac{1}{8\pi}(\nabla \times \boldsymbol{A})^2\right] \tag{1}$$

with the covariant derivative $D \equiv -\mathrm{i}\hbar\nabla - (e/c)\boldsymbol{A}$ and the local magnetic induction field $\boldsymbol{b}(\boldsymbol{r}, t) \equiv \nabla \times \boldsymbol{A}$. In this notation, two important characteristic length scales at

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temperature T are defined as follows [1]: the coherence length $\xi(T) \equiv \sqrt{\hbar^2/2m|\alpha|}$ and the magnetic penetration depth $\lambda(T) \equiv \sqrt{mc^2\beta/4\pi e^2|\alpha|}$. Here we simply assume that at all temperatures $\xi(T) \equiv \xi_0(1 - T/T_c)^{-1/2}$ and $\lambda(T) \equiv \lambda_0(1 - T/T_c)^{-1/2}$ with T_c being the superconducting transition temperature at zero field. As a result, the GL parameter κ becomes independent of temperature, that is $\kappa = \lambda_0/\xi_0$, and the upper and lower critical fields are obtained as $H_{c1}(T) = H_{c1}(0)(1 - T/T_c)$ and $H_{c2}(T) = H_{c2}(0)(1 - T/T_c)$, respectively, where $H_{c1}(0) = \phi_0 \ln \kappa/4\pi\kappa^2\xi_0^2$ and $H_{c2}(0) = \phi_0/2\pi\xi_0^2$ with the flux quantum ϕ_0 [1].

The TDGL equation is written down as [1]

$$D^{-1}(\partial/\partial t + \mathrm{i}e\phi/\hbar)\Phi = -\delta F/\delta\Phi^* \tag{2}$$

$$\sigma((1/c)\partial \boldsymbol{A}/\partial t + \nabla\phi) = -\delta F/\delta \boldsymbol{A} \tag{3}$$

where $(\partial/\partial t + \mathrm{i}e\phi/\hbar)$ is a covariant time derivative invariant under the local U(1) gauge transformation with a scalar potential ϕ, and Φ^* is a complex conjugate to Φ. Here D and σ are the normal-state diffusion constant and the conductivity, respectively. These quantities also have the relationship

$$4\pi\lambda(T)^2\sigma/c^2 = \xi(T)^2/12D \equiv t_0(1 - T/T_c)^{-1} \tag{4}$$

with

$$t_0 \equiv \pi\hbar/8k_B T_c. \tag{5}$$

Using the Maxwell equations, equation (3) is interpreted as follows: the total current density $\boldsymbol{j} \equiv (c/4\pi)\nabla\times\nabla\times\boldsymbol{A}$ is equal to the normal current $\boldsymbol{j}_n \equiv -\sigma(\partial\boldsymbol{A}/c\partial t + \nabla\phi)$ plus the super-current $\boldsymbol{j}_s \equiv (e/2m)[\Phi^*\boldsymbol{D}\Phi + \text{CC}]$, where CC denotes a complex conjugate to $\Phi^*\boldsymbol{D}\Phi$. These equations are supplemented with boundary conditions (BCs):

$$\boldsymbol{D}\Phi|_n = 0 \tag{6}$$

$$\nabla \times \boldsymbol{A}|_s = \boldsymbol{H}_e \tag{7}$$

where the index n in (6) denotes the normal direction on the boundary and the index s in (7) denotes the surface of the system. The BC (6) means that the system is surrounded by the insulator, while the BC (7) means that the vector potential on the boundary is determined by the external field $\boldsymbol{H}_e$.

Now we simulate equations (2) and (3) with BCs (6) and (7) for a type-II superconducting film by changing the external field stepwise. Here, we use a gauge in which the scalar potential is zero [5, 8] and we have also neglected the thermal noise. We consider a thin film in the x–y plane with a magnetic field applied along the z-direction, that is, $\boldsymbol{H}_e = H\hat{z}$, $\hat{z}$ being the unit vector along the z axis. Two variables, Φ and $\boldsymbol{A}$, are assumed to depend only on the coordinates x and y, and the third components of $\boldsymbol{A}$ and $\boldsymbol{D}$ are also neglected. This situation may be physically realized [8] if $\xi \ll d \ll \lambda$, d being the thickness of the film. In the following, we take the units of length to be ξ_0, time to be t_0, Φ to be $\Phi_\infty \equiv \sqrt{|\alpha|/\beta}$, H to be $H_{c2}(0)$, and A to be $\xi_0 H_{c2}(0)$. Under these conditions, we solve the TDGL equation on a square lattice with 128^2 lattice points. In our simulations we choose the time step to be 0.015 and the lattice spacing to be 0.5 (that is, $64\xi_0 \times 64\xi_0$ in physical units). We also set $T/T_c = 0.5$ and $\kappa = 2$ so that $H_{c1}(0.5\,T_c) = 0.04$ and $H_{c2}(0.5\,T_c) = 0.5$ in the present units. The initial state is taken to be a superconducting state $|\Phi| = 1$ and $\boldsymbol{A} = (0,0)$ without the external field at $T = 0.5\,T_c$ [8]. We will not discuss the numerical procedures any further in this letter, but see [5] and [8] for detailed discussion of them.

In the following, we simulate the magnetization process by stepwise change of the external field, H, with its increment $|\Delta H| = 0.05$. In the present simulations, the external field is changed as follows: first the field is increased from zero to 0.45, and then decreased from 0.45 to -0.45, and finally increased again to 0.45. Regarding the preceding state obtained at the field $H - \Delta H$ as a new initial state, we have run the simulation at each value of H for a rather longer period. The period is taken such that the system may be in the stable equilibrium state associated with each external field, or at least in the quasi-stable state which is different from the stable one in the arrangement of vortices, as is discussed later. In actual simulations, we set the period to be 10^5 steps. We have numerically checked that for each value of H the value of magnetic induction B after such long periods hardly alters. Here the magnetic induction B is obtained from the local induction field $b_z(\boldsymbol{r},t) \equiv \partial A_x/\partial y - \partial A_y/\partial x$ as $B \equiv (1/V)\int \mathrm{d}\boldsymbol{r}\, b_z$ with system volume V (128^2 in this case). We have also checked that the magnetic induction field at long periods penetrates into the system over the range of order λ from the boundary.

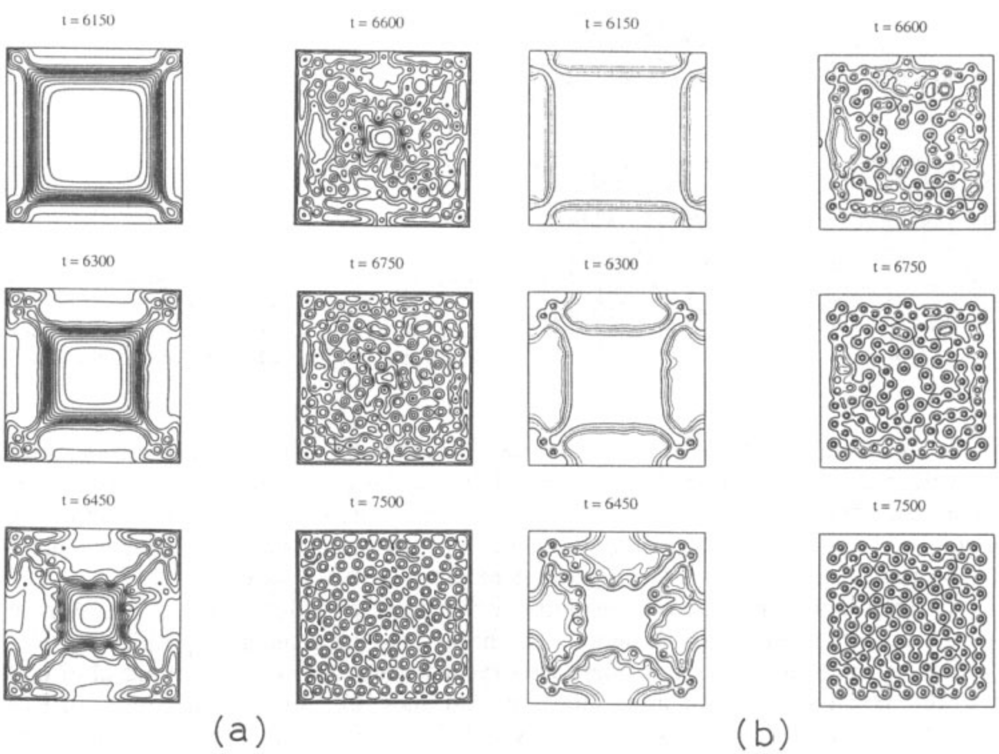

Figure 1. Time evolution of the spatial patterns of (a) b_z and (b) $|\Phi|$, after H is increased to 0.25 from 0.20 at $t = 6000$. The contour lines are shown with interval 0.02 for b_z and 0.2 for $|\Phi|$.

In figures 1(*a*) and (*b*) we show the time evolution of the local magnetic induction field b_z and the amplitude of the order parameter, $|\Phi|$, respectively, after the external field H is increased to 0.25 from 0.20 at $t = 6000$. In these figures the magnetic flux first penetrates from the boundary to form walls, and simultaneously destroy the superconducting state. Then, the dynamical instability of such flux walls with their coalescence occurs to create magnetic vortices. Finally, the vortex structure tends to be gradually rearranged toward the so-called Abrikosov lattice structure. Such a late stage process is very slow as was pointed out previously [5–8], and the lattice is not yet completely regular in this time scale, probably due to the finiteness of the system size and/or the simulation period.

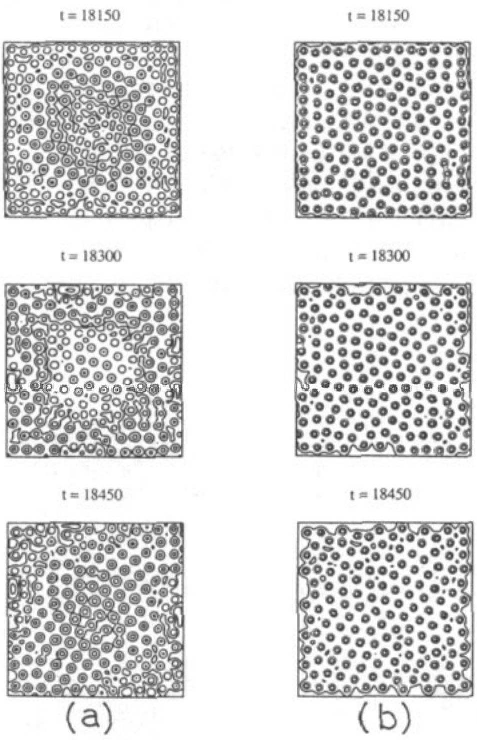

Figure 2. Time evolution of the spatial patterns of (*a*) b_z and (*b*) $|\Phi|$, after H is decreased to 0.25 from 0.30 at $t = 18\,000$. The contour lines are shown with interval 0.02 for b_z and 0.2 for $|\Phi|$.

In figures 2(*a*) and (*b*) we show the time evolution of b_z and $|\Phi|$, respectively, after H is decreased to 0.25 from 0.30 at $t = 18000$. In this case some of the vortices near the boundary have been pushed out of the system in the early stage according to the gradient of the induction field (figure 2(*a*)). Since then, the successive rearrangement of the remaining vortices is also observed to proceed very slowly.

In figure 3 we show the time evolution of b_z, after H is decreased to -0.20 from -0.15 at $t = 31500$. In this figure it is shown that the remaining positive vortices (that is, $b_z > 0$ shown by thick lines) are expelled from the boundary or are absorbed into the negative flux boundaries shown by thin lines [6]. Then, the negative vortices are immediately created, due to the flux wall instability, in just the position where positive vortices disappear.

In figures 4(*a*) and (*b*) we show the hysteresis loop of both the magnetic induction B and the magnetization $M \equiv (B - M)/4\pi$ against H, respectively. It has been

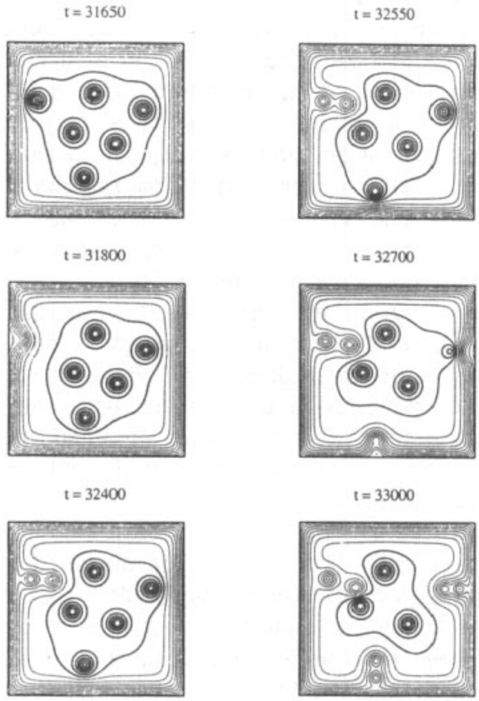

Figure 3. Time evolution of the spatial patterns of b_z, after H is decreased to -0.20 from -0.15 at $t = 31500$. The contour lines are shown with interval 0.02. Thick lines correspond to positive magnetic flux regions ($b_z > 0$), while thin lines correspond to negative ones ($b_z < 0$).

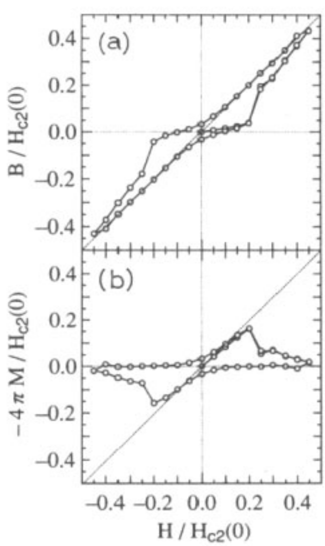

Figure 4. Hysteresis loop of (a) magnetic induction against external field (B–H) and (b) magnetization against external field (M–H). These curves are obtained for $\kappa = 2$ and $T = 0.5\,T_c$ so that $H_{c1}(0.5\,T_c) = 0.04 H_{c2}(0)$ and $H_{c2}(0.5\,T_c) = 0.5 H_{c2}(0)$.

shown that during one cycle of the field change a few vortices still remain even at $H = 0$ and thus the residual fields are indeed observed in this simulation. Note that similar results have also been obtained for $|\Delta H| = 0.01$. We have also found that with the increase of H from zero any vortices do not appear until 0.25, which is far beyond the value of the lower critical field (0.04 in this case). Similarly, any negative vortices are not observed until -0.20 with the decrease of H. These results may suggest the importance of specimen surface and also the existence of a surface barrier associated with flux penetration and expulsion, which was pointed out in [9]. The above values of H and also the magnitude of residual field, however, may be expected to depend on the details of simulation conditions such as ΔH, time period, system size and so on. Further simulations are needed changing these parameters as well as the physical quantities such as κ and T. The results of these simulations, as well as comparison with those of critical state models [1], equilibrium magnetization curves [2], and experimental data, will be published in the near future [10].

In summary, we have carried out several computer simulations of the isotropic, homogeneous TDGL equation for a type-II superconducting film in a magnetic field. We have demonstrated the dynamical processes of magnetization by changing the external field stepwise—such as magnetic flux penetration and its expulsion, and the formation and arrangement of vortex structures. We also have obtained the hysteresis curves on B–H and M–H diagrams. Although the present simulations have been done for only one parameter set and thus are still provisional, it is shown that the present direction of research may provide us with a useful tool in studying the dynamical behaviour of superconductors. Finally, we comment on our further investigations. To study the dynamics of recent high-temperature superconductors the present TDGL equation is inadequate and must be modified so as to include anisotropy, inhomogeneity, and Josephson interaction between layers as well as thermal noise [11, 12]. The present simulation method can be easily extended to such a system, as well as to a three-dimensional system. Moreover, the transport phenomena under the external current should be studied. These interesting problems are now under consideration.

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References

[1] Tinkham M 1975 *Introduction to Superconductivity* (New York: McGraw-Hill)
[2] Doria M M, Gubernatis J E and Rainer D 1990 *Phys. Rev.* B **41** 6335
[3] Garner J and Benedek R 1990 *Phys. Rev.* B **42** 6027
[4] Daemen L L and Gubernatis J E 1992 *Phys. Rev.* B **45** 314
[5] Enomoto Y and Kato R 1991 *J. Phys.: Condens. Matter* **3** 375
[6] Frahm H, Ullah S and Dorsey A T 1991 *Phys. Rev. Lett.* **66** 3067
[7] Liu F, Mondello M and Goldenfeld N 1991 *Phys. Rev. Lett.* **66** 3071
[8] Kato R, Enomoto Y and Maekawa S 1991 *Phys. Rev.* B **44** 6916
[9] Bean C P and Livingston J D 1964 *Phys. Rev. Lett.* **12** 14
[10] Enomoto Y, Kato R and Maekawa S 1992 in preparation
[11] Ullah S and Dorsey A T 1991 *Phys. Rev.* B **44** 262
[12] Dorsey A T and Fisher M P A 1992 *Phys. Rev. Lett.* **68** 694